

Observationes Cyclometricæ

by Adam Adamandy Kochański – Latin text with annotated English translation

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Translator's note: The Latin text of *Observationes* presented here closely follows the original text published in *Acta Eruditorum* [2]. Punctuation, capitalization, and mathematical notation have been preserved. Several misprints which appeared in the original are also reproduced unchanged, but with a footnote indicating correction. Every effort has been made to preserve the layout of original tables. The translation is as faithful as possible, often literal, and it is mainly intended to be of help to those who wish to study the original Latin text.

*ADAMI ADAMANDI E SOCIET.
JESU*

*Kochanski Dobrinniacy, Sereniss.
Poloniarum Regis Mathematici
& Bibliothecari, OBSERVATIONES
Cyclometricæ,
ad facilitandam Præxin accomodatæ;
ex Epistola ad Actorum Collectores.*

*BY ADAM ADAMANY FROM THE
SOCIETY OF JESUS*

*Kochański of Dobrzyń¹, Mathematician and
Librarian of the Most Serene King² of Poland, 5R
Cyclometric OBSERVATIONS,
accommodated for easiness of practical use;
from a letter to fellow readers of *Acta*.*

Qui Mathemata serio coluerit, nec tamen
ad difficillima quæque & adhuc insoluta Pro-
blemata vires ingenii sui pertentandas cen-
suerit, vix quenquam repertum esse existimo.
Haud equidem diffiteor, me quoque olim eo-
dem morbo laborasse, & ut alia præteream,
in Circulo quidem quadrando, vel examinan-
dis aliorum in eo conatibus, operæ non nihil
collocasse. Non attinet hic enumerare Metho-
dos, quas ea in re secutus fueram: unam tan-
tum, quam fortasse quispiam felicius excole-
re poterit, commemorabo. Persuaseram mi-
hi conjectura quadam, possibiles esse aliquas
Rectarum sectiones, quarum segmenta invi-
cem, & cum aliis rectis Longitudine vel Po-

I suppose one could hardly find anyone who
would seriously cultivate knowledge³ and who 10R
would nevertheless not think that strengths of his
talents are worth trying out on difficult and yet
unsolved problems. For my part, I do not deny
that I too was once affected by the same weak-
ness, and, to omit other things, I put not a small 15R
effort into squaring of a circle and in examination
of works of others attempting it. I does not be-
long here to list methods which I had followed in
this matter: I will mention only one, which per-
haps somebody luckier will be able to improve. I 20R
had convinced myself about a certain conjecture,
namely that certain sections of a straight line are
possible, whose fragments are incommensurable

¹Dobrzyń nad Wisłą – Kochański's birthplace, a town in Poland on the Vistula River, with settlement history dating back to 1065.

²John III Sobieski (1629 – 1696), from 1674 until his death King of Poland and Grand Duke of Lithuania.

³*Mathemata* could mean both knowledge or mathematics.

tentia incommensurabilia essent, Circuli tamen
 30 Areae, vel Peripheriae partibus Longitudine
 aut Potentia commensurarentur; ita ut
 inventa sectione istiusmodi, liceret ex ea Tetragonismum
 expedire Geometrice, vel saltem rationem Diametri
 ad Ambitum, in numeris ad lubitum maximis
 supputare.

Ad eam porro cogitationem videbar mihi non temere,
 35 sed illius Quadratricis, a Dinostrato inventae, ductu
 devenisse. At cum ab istis laboribus ad alia disparata
 studia animus avocaretur, illum tandem adjeci, & quidem
 magnorum Virorum exemplis incitatus, ad investiganda
 40 compendia quaedam Cyclometrica, Praxibus mechanicis
 utilia, idque tam in Numeris, quam Lineis; quorum
 nonnulla hoc loco adferre lubet.

to each other and to other straight lines in length
 and square, yet commensurable to parts of area 25R
 or circumference in length or square; so that by
 finding the section with this method, one might
 procure from it a quadrature of the circle geometrically,
 or at least compute the ratio of the diameter and
 circumference with as many digits 30R
 as one likes.

It seems that I have arrived to this idea not blindly,
 but guided by a quadratrix⁴, invented by Dinostratus⁵.
 And while my mind was diverted from this work by other
 separate pursuits, 35R
 eventually, inspired by examples of great men, I
 turned to investigation of certain profits pertaining to
 cyclometry, useful in mechanical practice, as much
 numerically as geometrically.

DIAMTERI AD PERIPHERIAM CIRCULI
 Rationes Arithmeticae⁶

	<i>Defectivæ</i>		<i>Excessivæ.</i>
A	1. ad 3. +—	Aa	1. ad 4. —
B	8. ad 25. +—	Bb	7. ad 22. —
Z	1.... 15.... 3.	Zz	1.... 16.... 3.
C	106. ad 333. +—	Cc	113. ad 355. —
Y	1.... 4697.... 3.	Yy	1.... 4698... 3.
D	530762. ad 1667438 +—	Dd	530875. ad 1667793. —
X	1.... 5448 ⁷ 3.	Xx	1.... 5449 ⁸ 3.
E	Diam. 2945 294501. Periph. 9252 915567 +—	Ee	Diam. 2945 825376. Periph. 9254 583360. —
V	1.... 14774.... 3.	Vv	1.... 14775.... 3.
F	Dia. 43 521624 105025. Per. 136 727214 560643 +—	Ff	Dia. 43 524569 930401 Per. 136 736469 144003 —

⁴Quadratrix of Hippias is a curve with equation $y = x \cot(\pi x/2a)$. It can be used to solve the problem of squaring the circle, although this is not a pure “ruler and compass” solution.

⁵Dinostratus (ca. 390 B.C. - ca. 320 B.C) was a Greek mathematician and geometer, a disciple of Plato.

⁶*Arithmetic ratios of diameter and circumference of a circle.* Ratios representing lower (“defective”) bounds are on the left, upper (“excessive”) bounds on the right.

⁷A misprint in the original text, should be 5548.

⁸Another misprint, should be 5549.

*Harum qædam minoribus terminis,*⁹

cc	$22\frac{3}{5}$	ad	71 —
d.	265381.	ad	833719 +—
e.	30685681.	ad	96401910 +—

Methodicam prædictorum Numerorum
 Synthesin in *Cogitatis, & Inventis Polyma-*
 45 *thematicis*, quæ, si DEUS vitam prorogaverit,
 utilitati publicæ destinavi, plenius exponam;
 sufficiet interim ad eorum notitiam insinuasse
 sequentia. Numeri Characteribus Z. Y.
 X. V. tam simplicibus, quam geminatis
 50 insigniti, sunt *Genitores*, e quorum ductu,
 Numeri illis subjecti C. D. E. F. simplici,
 geminoque caractere notati, procreantur
 hoc modo. Ratio 7. ad 22. Excessiva, ducta
 in Genitorem Z. 15; & adjecto ad Productum
 55 Diametri, numero 1. ad Peripheriæ autem,
 hoc altero adjacente 3; constituit Rationem
 C.106. ad 333, Defectivam: Genitor autem
 major Zz.16, ductus in eosdem terminos
 Excedentes 7. ad 22, adjectisque ad horum
 60 Producta numeris 1 & 3, conficit Rationem
 CC. 113, ad 355. Excess:

Similiter hi termini Excessivi 113, 355,
 multiplicati per Genitores Y.Yy. videlicet
 4697 & 4698. servata adjectione numerorum
 65 1. & 3 ad Producta Diametri Peripheriæque,
 offerent terminos Rationum D. & Dd, quæ
 longe propius accedunt ad Archimedeam, a
 Ludolpho, & Grümbergero nostro vastissimis
 expressam numeris. Eadem ratione in reli-
 70 quorum Terminorum genesi proceditur. Ut
 autem oculis ipsis usurpare liceat, quantum
 exactitudinis adferant Rationes illæ, visum
 est hoc loco adjicere Synopsin totius calculi,

I will explain the aforementioned method 40R
 more completely in *Polymathic thoughts and*
inventions, which work, if God prolongs my life,
 I have decided to put out for public benefit.
 In the meanwhile, for acquaintance with this
 method, the following introduction will suffice. 45R
 Numbers denoted by both single and double
 characters Z, Y, X, V are *Originators*, from
 which numbers subjected to them, denoted by
 single and double characters A, B, C, D, are
 derived this way. Ratio of 7 to 22, excessive, 50R
 is multiplied by the Originator Z. 15, and with
 added product of the diameter, equal to 1, and 3,
 close to circumference, yields the defective ratio
 of 106 to 333.¹⁰ Moreover, the major originator
 Zz.16, multiplied by the same exceeding bounds 55R
 7 and 22, and with numbers 1 and 3 added to
 the products, makes excessive ratio CC, 113 to
 355.¹¹

Similarly, those excessive bounds 113, 355,
 multiplied by originators Y and Yy, that is, 4697 60R
 and 4698, keeping addition of numbers 1 and 3,
 yield bounds¹² on the ratio D and Dd, which
 come far closer to the Archimedean ratio, ex-
 pressed by Ludolph¹³ and our Grümberger¹⁴ by
 great many digits. Remaining bounds are pro- 65R
 duced by proceeding in the same manner. In or-
 der to see accuracy of these ratios with one's own
 eyes, it seemed fit to add in this place synopsis
 of all calculations, by which the aforementioned
 ratios are tested against Archimedean ratio like 70R

⁹ Of these [ratios], some expressed in reduced form. Here, cc, d, and e are reduced forms of respectively Cc, D, and E, e.g. $\frac{71}{22\frac{3}{5}} = \frac{355}{113}$.

¹⁰ Fraction $\frac{22}{7}$ is transformed into $\frac{22 \cdot 15 + 3}{7 \cdot 15 + 1} = \frac{333}{106}$.

¹¹ This produces $\frac{22 \cdot 16 + 3}{7 \cdot 16 + 1} = \frac{355}{113}$.

¹² These bounds are $D = \frac{355 \cdot 4697 + 3}{113 \cdot 4697 + 1} = \frac{1667438}{530762}$ and $Dd = \frac{355 \cdot 4698 + 3}{113 \cdot 4698 + 1} = \frac{1667793}{530875}$.

¹³ Ludolph van Ceulen (1540 – 1610) was a German-Dutch mathematician who calculated 35 digits of π .

¹⁴ Christoph Grienberger SJ (1561 – 1636) was an Austrian Jesuit astronomer, author of a catalog of fixed stars as well as optical and mathematical works.

75 quo prædictæ Rationes ad Archimedeam, tanquam ad lapidem Lydium examinantur, ut appareat, quantum sit uniuscujusque peccatum, defectu vel excessu Peripheriæ taxato in partibus Diametri totius, in particulas Decimales subdivisæ.

against the Lydian stone¹⁵, so that it would become evident how big was the error of each of these ratios, with defect or excess of the circumference expressed as parts of the entire diameter, and with digits divided into small groups.

75R

*Examen Rationum Cyclometricarum.*¹⁶

Diam.	100000	00000	00000	00000	00000	Archimedis – Ratio.
Periph.	314159	26535	89793	23846	26433	
B.	312500	00000	Defectus ¹⁷			
	16519	26535				
Bb.	314285	71428	Excessus.			
	126	44892				
C.	314150	94339	62264	Defectus		
	8	32196	27592			
Cc.	314159	29203	53982	Excessus		
		2667	64189			
D.	314159	26535	81077	77120	Defect.	
			8715	46725		
Dd.	314159	26536	37862	02024	Excess.	
			48068	78178		
E.	314159	26535	89787	82814	Defect.	
			5	41031		
Ee.	314159	26535	89796	49172	Excess.	
			3	25326		
F.	314159	26535	89793	23833	89913	Defectus.
				12	36520	
Ff.	314159	26535	89793	23855	91866	Excessus.
				9	65432	

80 Ex hac Tabella colligitur: imprimis quantitas Defectus, vel Excessus cujusvis Rationis, taxata Fractione, cujus Denominator est Diameter supremo loco posita, videlicet 1 cum tot zeris, quot libet assumere: Numerator autem
85 erit is, qui in eodem cum Denominatore gradu Decimali consistit. Sic Rationis C Defectum metitur hæc Fractio $\frac{8}{100000}$ quæ exactior erit, si prolixior Denominator assumatur.

From this table one infers, first of all, quantities of the defect or excess of any ratio, estimated by a fraction whose denominator is the diameter placed in the initial position, with as many zeros as one wants to take. Numerator, on the other
80R hand, is this one, which takes the same position as the denominator. Thus the defect of the ratio C is estimated by the fraction $\frac{8}{100000}$, which would be more accurate if one took a longer denominator.

80R

85R

¹⁵Lydian stone (touchstone) – stone used to test gold for purity.

¹⁶*Examination of cyclometric ratios.*

¹⁷*Defect*, that is, the value of $\pi - B$. Similarly, *excess* (lat. excessus) is the value of $Bb - \pi$.

Colligitur ex eadem secundo. Rationes no-
 90 bis exhibitæ, adeo compendiosas esse, ut ea-
 rum nonnullæ, duplo pluribus notis Archime-
 deis æquivalent ; quanquam ipsa Cc duplum
 earum excedat, quæ proinde brevitate, nec
 95 non exactitudine sua, in Praxi cæteris præ-
 ferenda videatur, cui, dum quid accuratius
 quæritur, ipsa d succedat. Præter has quidem
 mihi suppetunt adhuc plures, consimili dote
 præditæ, sed eas, ne nimius videar, alteri oc-
 casioni servandas existimo. Concludam inte-
 100 rim singulari quadam, & ut ita dicam, curiosa
 Ratione, quæ est 991 ad $3113\frac{991}{3113}$, quæ cum
 Archimedeæ consentit in octonis notis priori-
 bus, ac tum primum illam incipit excedere,
 minus quam 23 centesimis.

From the same table, a second thing is in-
 ferred. Ratios exhibited by us are so advan-
 tageous, that some of them are equivalent to
 Archimedean ratio with twice as many digits as
 others; Yet Cc itself twice exceed others¹⁸, hence 90R
 in practice by shortness and accuracy it seems
 to be preferred to others. If one sought a more
 accurate one, d would be a successor. Besides
 those, I have at hand indeed even more of them,
 similar in quality to the mentioned ones, but, in 95R
 order not to appear excessive, I consider saving
 them for another occasion. I will, in the mean-
 while, conclude with a certain singular, and so
 to speak curious ratio, which is 991 to $3113\frac{991}{3113}$,
 which agrees with Archimedean in the first 8 dig- 100R
 its, and then it starts to exceed it, by less than
 23 hundredths.¹⁹

105 *GRAMMICÆ RATIONES*
CYCLOMETRICÆ,
Ad Usus Mechanicos.

Harum quidem complures olim a me repertæ;
 hoc tamen loco visum mihi est eam tantum
 110 proponere, quæ huic Anno præsentī, quo ista
 scribimus, affinitate quadam conjuncta est.

Oporteat igitur Semiperipheriæ B C D
 Rectam proxime æqualem reperire. Ducantur
 Tangentes B G, D H, quarum prior Radio AC
 115 æqualis, & jungantur GCH. Tum Radio CA
 secetur ex C arcus utrinque æquales CE &
 EF: quorum quivis complectetur Gradus 60,
 reliqui autem BE, DF singuli gr. 30. Agatur
 per E Secans AI, determinans Tangentem BI.
 120 Capiatur tandem HL, æqualis Diametro BD;
 ac tum ducatur IL.

GEOMETRIC CYCLOMETRIC
CONSTRUCTIONS,
For Use by Mechanics. 105R

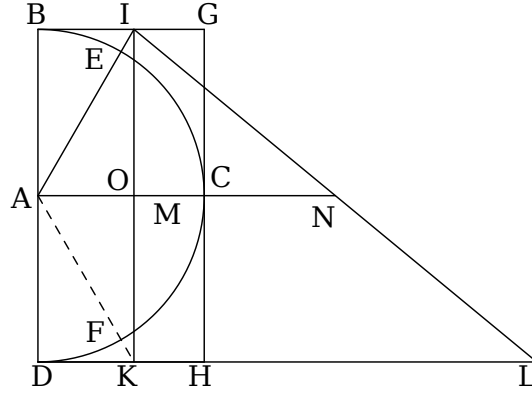
Of which several were once found by me. In
 this place, nevertheless, it seemed appropriate
 to present only one, associated with the current
 year, in which we write this.

It would be then required to find a straight 110R
 line nearly equal to the semicircle BCD. Let tan-
 gent lines BG, DH be drawn, equal to the radius
 AC and connected by GCH. Then from C, let
 both parts of the arc be cut by CE and EF²⁰,
 equal to the radius CA. Each of them will em- 115R
 brace the angle of 60 degrees, while the remain-
 ing angles BE, DF will be 30 degrees each. Let a
 line AI be driven through E, determining the ex-
 tent of the tangent BI. Finally, let HL be taken
 equal to the diameter BD; and then let IL be 120R
 drawn.

¹⁸That is, it exceeds B, Bb, and C in accuracy.

¹⁹Defining $r = \frac{3113\frac{991}{3113}}{991} = 3.1415192677\dots$, we have $r - \pi \approx 0.23 \cdot 10^{-7}$.

²⁰This should likely be CF.



Dico Inprimis IL æqualem esse Semiperi-
pheriæ BCD proxime. Demonstratur calculo
Trigonometrico. Intelligatur autem ducta es-
se IK, quæ Tangentes BI, DK conjungat.

Quoniam ad Radium

AB. 100000 00000 00000.

Tangens gr. 30 est

BI. 57735 02691 89626. Erit hujus

130 Compl. ad Radium, ipsa

IG. 42264 97308 10373. Igitur

Tota KH ✕ HL, sive

KL. 2 42264 97308 10373. Ergo

IK q ✕ XL q.

135 9 86923 17181 95572 75995 52843 99129.

Horum Radix est.

IL. 3 14153 33387 05093. Hæc autem

Deficit ab Archimedeæ.

Z. 5 93148 84700.

140 Continetur in AB. vicib.

X. 16859.

Dico deinde, Peripheriam sic inventam, ab
Archimedeæ veræ proxima deficere minori Ra-
tione, ea, quam habet Unitas ad Decuplum
currentis Anni 1685 a Christo nato, Æra vul-
gari numerati; majore autem, quam eadem
habeat ad decuplum anni 1686, proxime secu-
turi. Cum enim Peripheria nostra IL, ab Ar-
chimedeæ (præcedentis Tabulæ) deficiat nu-

I say in the first place that IL is nearly equal
to the semicircle BCD. This is demonstrated by
trigonometric calculations. Let us assume that
the line IK is drawn, which connects tangents
BI, DK. Since, with the radius

125R

AB. 100000 00000 00000,

tangent of 30 degrees is

BI. 57735 02691 89626.

Its complement²¹ to the radius, IG itself, will be

130R

IG. 42264 97308 10373. Therefore,

together KH ✕ HL, or

KL. 2 42264 97308 10373. Hence

IK q ✕ XL²² q.

9 86923 17181 95572 75995 52843 99129,

135R

of which the root is

IL. 3 14153 33387 05093.²³ But this is

short of Archimedean by

Z. 5 93148 84700,²⁴

contained in AB approx.

140R

X. 16859.²⁵

I say then that the circumference found this
way differs from the Archimedean ratio by less
than the ratio of one to ten times the date of
the current year 1685 after Christ, numbered
with the common era, and more than one to ten
times the date of the next year to come, 1686.
When indeed our periphery IL is short of the
Archimedean (of preceding table) by the number

145R

²¹ $1 - \tan 30^\circ$.

²²Obviously a misprint, should be KL instead of XL. IK q ✕ KL q. means $IK^2 + KL^2$.

²³ $IL = \sqrt{4 + \left(3 - \frac{\sqrt{3}}{3}\right)^2} = \frac{1}{3}\sqrt{120 - 18\sqrt{3}}$.

²⁴ $Z = \pi - IL \approx 0.0000593148847$.

²⁵ $X = \frac{1}{Z} \approx 16859$.

150 mero Z, qui totam Diametrum, numero AB
taxatam, metitur numero X: manifestum est,
Unitatem ad numerum præsentis Anni decu-
plum, videlicet 16850, majorem habere ratio-
nem quam ad X, priore majorem: minorem
155 autem, quam ad hunc 16860, per demonstra-
ta Prop. 8 Quinti Elem. Euclid. E quibus
Praxeos istius ἐνπορία καὶ ἀκρίβεια, cum
intellectu comprehendī, tum memoria facile
retineri poterit.

160 Epimetri loco adjungam alteram Praxin
Linearem Mechanicorum Circino opportunis-
simam, quod ea continua Diametri bisectione
peragatur, sitque longe exactior præceden-
te: sic autem instituitur. Dati Circuli Dia-
165 metrum, Circino bisectioni destinato, divide
in partes 32. Talium enim Peripheria erit
 $100\frac{17}{32}$, hoc est, erit eorum Ratio, 1024. ad
3217. In Praxi igitur, Triplo Diametri, sive
partibus 96, adjiciendæ erunt $\frac{4}{32}$, sive $\frac{8}{1}$ to-
170 tius Diametri, & insuper Semis unius Trigesimæ
secundæ, cum alterius Semissis particula
decima sexta. Hujus ἐγχειρήσεως exactitudinem
probat calculus, quo provenit Periph-
eria P. 314160156.– quæ Archimedeam ex-
175 cedit numero Q891, qui minor est Defectu
Z, Peripheriæ præcedentis. Quamquam nec
istud subticendum sit, istam Praxin in Ma-
joribus Circulis potissimum locum habere, in
parvis oculorum effugere, quoad particulam,
180 postremo addendam.

Z, which measures the whole diameter, expressed 150R
by the number AB, with the number X: it is clear
that the ratio of the unity to ten times the cur-
rent year, or 16850, is larger than the ratio of 1
to X, and less than 1 to 16860, as demonstrated
by prop. 8 of the 5th book of Euclid²⁶. From 155R
this exactness but also easiness of the construc-
tion, when embraced by the intellect, can easily
be retained in memory.

As a supplement, I will add another linear
construction suitable for the compass of mechan- 160R
ics, which would be carried out by successive bi-
sections of the diameter, and would be far more
exact than the previous one: it is set up as fol-
lows. Given the diameter of the circle, intended
for the bisection by compass, divide it into 32 165R
parts. Of such kind, the circumference will be
 $100\frac{17}{32}$, that is, will be the ratio of 1024 and 3217.
In practice, therefore, to three diameters, or 96
parts, $\frac{4}{32}$ will be added, or $\frac{8}{1}$ of the total diam-
eter²⁷, and, moreover, one half of 32nd, with $\frac{1}{16}$ 170R
of another particle's half.²⁸ Calculations prove
exactness of this procedure, yielding Circumfer-
ence P. 3.14160156, which exceeds Archimedean
by the number Q891, which is smaller than
the defect Z, of the preceding circumference²⁹. 175R
In spite of this, one must not be silent about
the fact that this construction has its place prin-
cipally applied to larger circles, yet in smaller
circles it is beyond one's ability to see, especially
with respect to the small particle added at the 180R
end.

References

- [1] Euclid. *The thirteen books of the Elements*, volume 2. Dover, New York, 1956.
- [2] Adam Adamandy Kochański. Observationes cyclometricae ad facilitandam praxin accomodatae. *Acta Eruditorum*, 4:394–398, 1685.

²⁶Prop. 8 of Euclid's book 5 says: *Of unequal magnitudes, the greater has to the same a greater ratio than the less has; and the same has to the less a greater ratio than it has to the greater.*[1]

²⁷Clearly, the author means 1/8 here.

²⁸This amounts to $\frac{96}{32} + \frac{4}{32} + \frac{1}{2} \cdot \frac{1}{32} + \frac{1}{32 \cdot 32} = \frac{3217}{1024} = 3.1416015625$

²⁹ $Q = \frac{3217}{1024} - \pi \approx 0.00000891$.